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Abstraction, Pragmatism, and History in Mach's Economy of Science

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Introduction

Ernst Mach's appeal to the 'economy of science' has been interpreted as an overarching principle of minimization (Stein 1967, 1977), promoting the increasing simplification of scientific knowledge via principles that increase calculating power without adding substantively to the knowledge embedded in empirical facts. There is a growing literature (Gori, Banks, Pojman, Wulz, Serra, and Maia) arguing for a more robust understanding of Mach's 'economy of science'. Machian 'economy' appeals to the continuity between scientific experiences and concepts, but also to the increasing complexity of scientific concepts, building on connections between what Mach called world-elements or sensation elements (Banks 2003). Mach's account emphasises not only continuities between experiences that allow for simplification, but also areas of divergence that promote the branching of scientific concepts and methods. I emphasise the roles of abstraction, pragmatism, and history in Mach's economy of science, and I argue that these elements allow Mach to investigate the productive tension between creative and conservative moments in the history of science.[1]

Mach's World-Elements

Ernst Mach is one of the nineteenth-century scientists, like Hermann von Helmholtz, whose work is not easy to fit into contemporary disciplinary categories. He is not merely a physicist, physiologist of perception, or philosopher of mind. Certainly, Mach considered himself to be a physicist, but he

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[1] While this line of thought is of course inspired by Kuhn, I am not suggesting (here) that it inspired him.

would have wished his work in physiology and psychology, among other areas, to be recognised. Erik C. Banks (2003) reads him as fitting into the tradition of 'natural philosophy', which included physics, mathematics, what we would now call philosophy of mind, theology, and the life sciences.

Still, Mach can be placed in the nineteenth-century tradition of '*Erkenntnistheorie*' or theory of knowledge, in which physiology, physics, and the life sciences worked in interaction. That tradition was increasingly responsive to work in sense physiology and what is now called biology. But it was also affected by debates over historical methods and over philosophical approaches, including Kantianism, materialism, and empiricism (Patton 2004, chapters 1 and 2).

The account that follows emphasises Mach's approach to a central concept in his scientific reasoning: the economy of science. There is a productive tension in Mach's work between the creative moment in the history of science, in which scientists can introduce principles or concepts that make new connections between experiences, and the minimisation project that Mach took equally seriously, of explaining experiences and reasoning economically using accounts of our physical and physiological interaction with the environment. Both the creative and the minimisation projects turn our attention to Mach's account of the distinction between the subjective and objective in epistemology. I will argue that Mach's overarching concern, in accounting for the role of the scientist in the economy of science, is with the *pragmatic history* of the experiencing and creative knower, rather than with exclusive, reductive phenomenological or biological explanations.

Debates over Mach's work in the 1960s, 1970s, and 1980s centred on a number of key issues: Mach's dispute with Ludwig Boltzmann over the 'reality' of atoms and his remarks about the epistemological foundations of Newtonian space and time (including the well-known 'bucket' and 'two spheres' thought experiments).

John Blackmore's copious publications in the 1970s and 1980s followed on V. I. Lenin's earlier reading of Mach as a phenomenalist (Lenin 1909/1972). Ivor Grattan-Guinness (1974) observes in a review of Blackmore's *Ernst Mach* that Blackmore opposes 'representationalism', which 'asserts that the real world is unattainable and represented to us only through mental appearances', to Mach's supposed 'presentationalism', which 'advocates that the external world *is* the appearances we receive of it' (Grattan-Guinness 1974, p. 76, emphasis in original).

The latter seems to be Blackmore's sense of 'phenomenalism': that the external world of sense is not a representation of something beyond sensed particulars, but rather the appearance just *is* the reality. Blackmore supports the 'representationalist' view, that the appearances are indications or signs of an underlying reality:

> To clarify and occasionally criticise Mach's philosophical ideas especially those of a phenomenalistic or Buddhistic drift, I have often contrasted his

> point of view with what I call 'common sense'. I mean the representationalistic view of Galileo, Boyle, Locke, and Newton – ideas widely accepted and used by practical people today and the only epistemology compatible with a reasonable understanding of the process of perception as accepted by most scientists in the field.
>
> (Blackmore 1972, pp. x–xi)

Along with other critics, Seaman (1975) is sceptical of Blackmore's claim that science and common sense converge on representationalism, and he questions Blackmore's assertions that Mach is a phenomenalist and that his positions are in conflict with any 'reasonable understanding of the process of perception' by either 'scientists in the field' or 'practical people'.

Banks' first book, *Ernst Mach's World Elements* (2003), presents a reading of Mach that does not interpret him as a phenomenalist, and thus avoids Blackmore's criticisms. Banks' reasoning is subtle. He builds a reading of Mach on two fronts that are relevant here:

(1) A detailed account of Mach's understanding of the process of perception, which leads to
(2) A detailed account of Mach's 'neutral monism', which Banks argues is not equivalent to or reducible to phenomenalism.

Banks' second book, *The Realistic Empiricism of Mach, James and Russell* (2014), follows up on the second point in detail, presenting Mach as an early supporter of neutral monism along with James and Russell. Scott Edgar (2013) explains how Mach's neutral monism allows him to deny coherently the existence of things in themselves, a problem also faced by F. A. Lange, who found a less consistent response. A key point that Edgar raises is that Mach's denial of things in themselves reaches to denying an absolute *subject* as well:

> In rejecting the view that there are any permanent substances or things in themselves underlying the sensation-like elements of experience, he rejects the idea of a permanent subject or mind that contains sensations, just as he rejects the idea of permanent bodies underneath our sensations of objects in the world. Thus for Mach, these sensation-like elements are strictly speaking neither mental nor physical considered in themselves. Rather, he thinks we consider an element a physical property when we regard it as part of a complex that represents a 'body' and we consider it a sensation when we regard it as part of a complex that represents an 'ego'.
>
> (Edgar 2013, p. 113, note 11)

The upshot of new readings of Mach from scholars including Banks, Edgar, Alix Hui (2013), and Richard Staley is to reject the idea that Mach was a naïve phenomenalist about perception and to defend a more complex picture of Mach as a neutral monist. To Mach, perception presents us with 'world-elements' that are neither exclusively objective nor exclusively subjective.

Any attempt to reduce one set of elements to another, or to show that we can make inferences from 'subjective' perceptions to a realm of 'real', 'objective' underlying objects, is ruled out. This is not because Mach is committed to phenomenalism – it is because he is committed neither to phenomenalism nor to representationalism.

Mach explains that we use world-elements to construct a world by analysing our sensations, but we do not know a priori whether those sensations are 'mere' phenomena (i.e. merely a response of our sensorium to stimuli) or whether they are indications of external objects. Banks points out that, to Mach, most of our sensations are both. As I would gloss it, sensations include information about the 'object' of perception – which is not mind-independent, but rather what is objective *in* perception – and information about the 'subject' of perception, 'direct' information about which is not accessible independently of perception either.[2]

The object-directedness of perception, for Mach, is not something we can *know about* immediately. This is not the position that we must construct a priori concepts of objects in order for perception to be object-directed: Mach rejects that position definitively in the introduction to *The Analysis of Sensations*. Rather, it is the position that perception is not given to us as object-directed: we cannot know immediately what in our perceptions, as we experience them, is objective and what is subjective.

To Mach, the physiology of perception can establish one perspective on the contents of perception. That perspective allows us to distinguish those parts of perception that are *due to* the process of perception and those parts that are, for instance, introduced by measurement processes. Mach distinguishes carefully (e.g. in his 1903 essay 'Space and Geometry from the Point of View of Physical Inquiry') between metric and physiological space.

If sensation is to contribute to the depth of scientific knowledge, it must be able to be managed as part of the economy of science. For Mach, this means first of all that experienced sensation must be analysable, in the classic, chemical sense that sensation can be divided into distinct parts of recognisable types. Mach uses techniques from physics and psychology, especially the theory of manifolds, to make this explicit (Mach 1903).

Objectivity and History in Mach and Helmholtz

Mach's contemporary Helmholtz also thinks that whether sensations allow for inferences about subjective or objective content is a scientific matter that should be investigated empirically, not a determination that can be made on

[2] See Banks (2003) for detailed historical accounts.

the basis of conceptual analysis alone.[3] But Helmholtz argued that we must assume an a priori causal law that describes the stable relationships between external objects, conceived as the sources of our sensations and representations, and the physiological constitution and perceptual system of the subject (Hatfield 2018; Patton 2018).

Mach does not base his system on an a priori law of causality as Helmholtz does. Helmholtz's reliance on causal reasoning and on induction is his way of securing the stability and justification of inference from perceptual experience. Helmholtz argues that perceptions are signs, and not copies, of the external objects that are their sources, a position Brian Tracz has dubbed 'Ignorance' (Tracz 2018, p. 64): 'In Helmholtz's terms, "our images of the things in our representation are not similar to their objects" [Helmholtz 1867, § 2.3, pp. 590]. And since Helmholtz maintains that we can only obtain knowledge of objects through perception, this fundamental dissimilarity leaves us ignorant, in some respect, of the things we represent' (Tracz 2018, p. 64).

But Helmholtz maintains that we can find what is true in our representations by finding the causal laws that describe the actual relationships between perceptual signs and what they depict (De Kock 2014, 2015):

> Each law of nature states that on preconditions that are similar in certain respects consequences that are similar in certain other respects always occur. Since similars in our sensible world are indicated by similar signs, the nomological sequence of similar effects following similar causes corresponds to an equally regular sequence in the realm of our sensations.
>
> (Helmholtz 1878, p. 13)

Helmholtz argues that inferences from previous experience can influence present experience as it occurs. Thus, the *history* of our perceptual experience becomes important for him:

> . . . the remembered images from earlier experiences work together with present sensations to bring forth an intuitive image, which intrudes upon our power of perception with compelling force, without what is given through memory and what is given through present perception being separated in consciousness.
>
> (Helmholtz 1867, § 26, pp. 436–437, my translation)

Helmholtz distinguishes between the inductive inferences and prior perceptual images that can be operative in occurrent perception and the causal laws that he sees as underlying veridical inferences from perception.

While Mach also emphasises the role of history in the growth of scientific knowledge, he does not make this distinction. For Mach, the laws of nature

[3] See Patton (2018), including references to other work.

and other 'economical' scientific methods replace pre-theoretical 'instinctive knowledge'. But scientific methods and principles arise out of instinctive knowledge. The section following will explain Mach's concept of instinctive knowledge and its role in his account of the development of basic laws of mechanics: to obviate reliance on a priori elements.[4]

Instinctive Knowledge and the Laws of Nature

Mach, like Helmholtz, argues that prior experience affects the way we behave in ordinary life and in scientific research. Mach's position can be found in his book *The Development of Mechanics*.[5] There, Mach argues that previous perceptual experience impinges upon current experience. While Helmholtz focuses on the *lack* of resemblance between our perceptions and the external objects that are their sources, Mach emphasises that observations are indications of 'the processes of nature':

> How does instinctive knowledge originate and what are its contents? Everything which we observe in nature imprints itself uncomprehended and unanalysed in our percepts and ideas, which, then, in their turn, mimic the processes of nature in their most general and most striking features. In these accumulated experiences we possess a treasure-store which is ever close at hand and of which only the smallest portion is embodied in clear articulate thought.
>
> (Mach 1919, p. 28)

A common feature of Mach's and Helmholtz's work is the focus on *series* or *sequences* of observations, as opposed to *singular* perceptions. This feature distinguishes their work from that of philosophers who emphasised the role of singular observations in 'confirming' hypotheses about external objects or processes. Mach does identify singular aspects of perceptions, but only in the context of series or sequences. These sequences have patterns, features that 'mimic the processes of nature in their most general and most striking features'.

One of the earliest chapters of *The Development of Mechanics* focuses on the role of 'instinctive knowledge' in Simon Stevin's derivation of the principle of the inclined plane, 'a veritable acme' of Stevin's career.[6] Stevin, a Flemish mathematician and engineer, performed thought experiments on the 'wreath

[4] I am grateful to Thomas Uebel for clarification of this passage.

[5] Mach (1919, first edition published in 1883). The title chosen for the English translation of the book was unfortunately, and wrongly, *The Science of Mechanics*.

[6] Dijksterhuis (2012/1970, p. 52). Serra et al. (2018) discuss Mach's analysis of instinctive knowledge in detail, including previous analyses by Pojman, Feyerabend, and Buchler (§ 2).

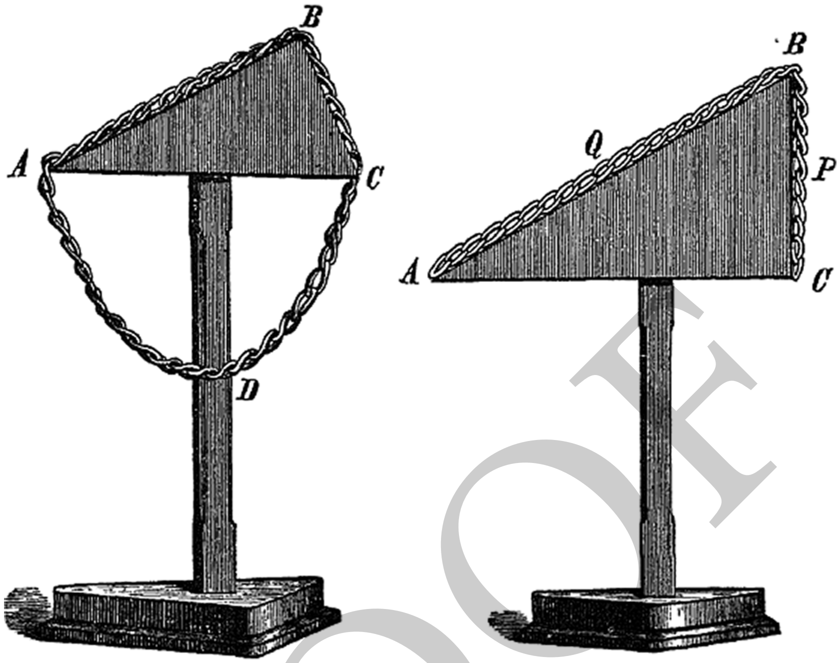

Figure 8.1 Left: Stevin's inclined plane with endless chain (Mach 1919, pp. 24–25). Right: Stevin's plane with the symmetrical portion of the chain removed (Mach 1919, pp. 24–25)

of spheres' or 'clootcrans' (figures 19 and 20 in Stevin 1586, p. 41, discussed in Mach 1919, pp. 24–25) (see Figure 8.1).

As Mach summarises his reasoning,

> He imagines a triangular prism with horizontally placed edges, a cross-section of which ABC is represented in Fig. 19. For the sake of illustration we will say that AB=2BC; also that AC is horizontal. Over this prism Stevinus[7] lays an endless string on which 14 balls of equal weight are strung and tied at equal distances apart. We can advantageously replace this string by an endless uniform chain or cord. The chain will either be in equilibrium or it will not. If we assume the latter to be the case, the chain, since the conditions of the event are not altered by its motion, must, when once actually in motion, continue to move for ever, that is, it must present a perpetual motion, which Stevinus deems absurd. Consequently only the first case is conceivable. The chain remains in equilibrium. The symmetrical portion ADC may, therefore, without disturbing the equilibrium, be

[7] Mach calls Stevin 'Stevinus'.

> removed [figure 20]. The portion AB of the chain consequently balances the portion BC. Hence: on inclined planes of equal heights equal weights act in the inverse proportion of the lengths of the planes.
>
> (Mach 1919, pp. 24–25)

Dijksterhuis summarises Stevin's theorem as follows:

> Proposition XIX. Given a triangle, whose plane is at right angles to the horizon, with its base parallel thereto, while on each of the other sides there shall be rolling spheres, of equal weight to one another: as the right side of the triangle is to the left side, so is the apparent weight of the spheres on the left side to the apparent weight of the spheres on the right side.
>
> (1970/2012, p. 52, trans. corrected)

'Apparent weight' is a translation of the term '*Stallwicht*', 'meaning the component of an acting force which is actually exerting an influence' (Dijksterhuis 1970/2012, p. 52). Dijksterhuis notes:

> The admiration we owe to Stevin's wreath of spheres need not blind us, however, to the fact that the intuition on which it rests is not equalled in value by the logical force of the reasoning. The salient point of the demonstration obviously consists in the impossibility of a perpetual motion. Now the whole contrivance is conceived in the ideal realm of rational mechanics, where all disturbing influences, such as friction and air resistance, are believed not to exist. But in this realm a perpetual motion is by no means impossible: a simple pendulum, which has been pulled to one side and then released, forms an example of it. The wreath of spheres would indeed perform a perpetual motion if it were given an initial velocity.
>
> (Dijksterhuis 1970/2012, p. 54)

In *The Development of Mechanics*, though, Mach explains Stevin's reasoning quite differently. Dijksterhuis focuses on the question of whether 'the ideal realm of rational mechanics' allows for the possibility of a perpetual motion. Mach argues, instead, that Stevin is appealing to the 'instinctive knowledge' of the people Stevin wishes to persuade with his derivation of the theorem. In particular, Mach argues that Stevin is appealing to our lack of experience of a perpetual motion of the kind required to refute the law of the inclined plane:

> He feels at once, and we with him, that we have never observed anything like a motion of the kind referred to, that a thing of such a character does not exist . . . we accept the conclusion drawn from it respecting the law of equilibrium on the inclined plane without the thought of an objection, although the law if presented as the simple result of experiment . . . would appear dubious.
>
> (Mach 1919, p. 26)

Mach concludes that Stevin places 'instinctive knowledge' above simple observations made in experiment. The *basis* of the instinctive knowledge in question is a sequence of observations, and so we might wonder how Mach makes this distinction. Mach goes on to say:

> That Stevin ascribes to instinctive knowledge of this sort a higher authority than to simple, manifest, direct observation might excite in us astonishment if we did not ourselves possess the same inclination . . . We feel clearly, that we ourselves have contributed nothing to the creation of instinctive knowledge, that we have added nothing to it arbitrarily, but that it exists in absolute independence of our participation. Our mistrust of our own subjective interpretation of the facts is thus dissipated.
>
> (Mach 1919, p. 26)

There are two salient points about Machian 'instinctive knowledge' that should be emphasised. First, it is not a 'subjective interpretation', but rather based on autonomous, independent facts. That does not mean that it has no subjective component, in the sense discussed in the sections above. Rather, it means that our *interpretation* of the observations involved does not rest on grounds exclusive to the subject, but rather on grounds independent of the subject's participation.

Second, Mach's instinctive knowledge rests not on *single* observations, but on *sequences* of observations that express relations between *groups* of facts. In another context, Mach writes along the same lines of the more formal knowledge of physics, represented in equations[8]:

> [W]hat we have found is that between the [elements] of a group of facts . . . a number of equations exists. The simple fact of change brings it about that the number of these equations must be smaller than the number of the [elements]. If the former be smaller by one than the latter, then one portion of the [elements] is uniquely determined by the other portion.
>
> (Mach 1986, pp. 180–181)

As Banks emphasises, the method of differences is fundamental to Mach's method in science:

> The tension between the elements and their ordering into this general manifold of space, time, and matter is a general problem in Mach's philosophy of nature. The divide falls between his Heraclitean view that the elements are transitory unique events, arising and vanishing and possessing always an individual existence, and his view that space, time, and matter, however unreal they may be on a fundamental level, represent for Mach economical permanencies that must be acknowledged as a task

[8] I am grateful to Richard Staley for clarification of this passage.

> of science . . . Mach said that he considered the real facts of nature to be the existence of 'differences' or inequalities . . . Mach's elements are the differences of state in the world and, by a careful tracking of their effects on one another, the determinations of the rates and magnitudes of those effects, Mach thought one could deduce the existence of independent potential sources and relations of intensity from this raw data by finding orderings in it.
>
> (Banks 2003, p. 239)

Mach's account is based on the following principles:

- The world is constructed from functional connections between world-elements.
- Different functional connections between those world-elements can bring different states of affairs into view. These functional connections may draw on previous experience.
- Features of experience, such as its *continuity* and whether we can provide a coherent *history* of our experience in giving explanations, support efficient orderings of the data of experience.
- Anti-reductionism: there is no one correct or accurate state of affairs to which a complex of world-elements must be reduced. There can be multiple accurate expressions of the content of a complex of world-elements.

These principles form the core of Mach's economy of science. In the section that follows, I will present a dynamical account of this economy and provide an example of how it works.

The Dynamics of the Economy of Science

Following on the earlier work of Paul Feyerabend (1987, 1999), Paul Pojman (2000, 2011), and Banks (2004), there has been a recent resurgence of interest in Mach's economy of science, including work by Pietro Gori (2018, 2019), Isabel Serra and Elisa Maia (2018), Monika Wulz (2015), and myself (2018). The more recent scholarship focuses on Mach's economy as a subject of interest in itself. Some earlier work (Stein 1967, 1977) was more interested in the question of why Mach seemingly dismissed Newton's methods. On the other hand, work on Mach's empirically founded research by Don Howard, especially, has made much of the differences between Mach and the neo-Kantians and other apriorist thinkers.

In *The Development of Mechanics*, Mach uses the words 'economy' and 'economical' in a number of ways, all of which contribute to his picture of science as a system. I have gathered his uses into Table 8.1 (Patton 2018).

The *methods* on the left side of Table 8.1, which we will discuss in a moment, contribute to the economy of science. The *features* of science on

Table 8.1 *Typology of the economy of science*

Methods that promote economy	The effects of these methods
'Instinctive knowledge'	Transparency
The method of differences	Empirical fruitfulness
The law of continuity of experience	The ability to disregard details
Relations of the whole	Computational power
	Minimisation
	Completion of experience

the right side of Table 8.1 are desirable results of the employment of those methods. One common interpretation of Mach on the economy of science is to read him as arguing in a simplistic way that 'economy' adds up to minimisation, or increased computational power, alone – and certainly, Mach does say that economy involves minimisation. But that statement is made in the context of *other* statements that require the economy of science to include *every aspect* of science in its methods, to focus on the *completeness* and *continuity* of the experience that lies at the basis of science, and to conceive of every scientific result within the whole of knowledge (*Erkenntnis*).

Here, it will be instructive to examine another of Mach's discussions of 'instinctive knowledge': his critique of Archimedes on the law of the lever, discussed early in the twentieth century by Giovanni Vailati, and recently by Paolo Palmieri (2008) and Maarten van Dyck (2009).

As Mach observes, Archimedes begins from a form of 'instinctive' knowledge. He starts with two propositions that Archimedes takes to be self-evident:

a. Magnitudes of equal weight acting at equal distances (from their point of support) are at equilibrium.
b. Magnitudes of equal weight acting at unequal distances (from their point of support) are not in equilibrium, but the one at the greater distance sinks. (Mach 1919, pp. 8–9)

As Mach notes, the first proposition can be supported by a form of instinctive knowledge. If a 'spectator' looks at an experimental set-up with two masses of equal weight and at equal distances from the fulcrum of a balance, we can perceive, 'determined by the symmetry of our own body', that they will be in equilibrium, and this will be 'a highly imperative *instinctive* perception' (Mach 1919, p. 10, emphasis in original). Thus far, Archimedes' derivations are similar to Stevin's, examined above. Just as we instinctively perceive that no perpetual motion will occur in Stevin's wreath of circles, we also perceive that two weights will not move if they are equal and placed at equal distances from the fulcrum of a balance (assuming a proper set-up).

Mach goes on to say that Archimedes attempts to *demonstrate* the law of the lever from a process of reasoning that ultimately fails. As van Dyck puts it,

> Both the spirit and content of Mach's criticism are best captured by his well-known rhetorical question about how it would be possible to derive 'by speculative methods' the inverse proportionality of weight and lever-arm 'from the mere assumption of the equilibrium of equal weights at equal distances'. Mach's answer is ... that Archimedes' rational argument ... presupposes what needs to be proved. He locates the exact step in the argument where this happens, in what Palmieri calls the equilibrium-preserving assumption. This assumption basically states that equilibrium is not disturbed if one of the equilibrating weights is replaced with two weights placed symmetrically around the original weight, each having half the weight of the original body.
>
> (van Dyck 2009, pp. 315–316)

Mach argues that 'Archimedes pursued exactly the same tendency as Stevinus, only with much less good fortune' (Mach 1919, p. 27). This 'tendency' was to formalise the reasoning from 'instinctive knowledge' that pre-theoretical reasoners tended to apply. The difference between Archimedes and Stevin, to Mach, is that Stevin appeals to instinctive knowledge in his proof of a proposition that is properly supported by that knowledge. Archimedes, on the other hand, appeals to a general proposition – *not* instinctive knowledge – in his attempt to support another general proposition. Thus, Archimedes tries to prove that a proposition (the law of the lever) is applicable generally by appealing to a general principle, and not by showing that it is reducible to a form of instinctive knowledge that rules out error. As Mach puts it:

> [W]hen Archimedes substitutes for a large weight a series of symmetrically arranged pairs of small weights, which weights extend beyond the point of support, he employs in this very act the doctrine of the centre of gravity in its more general form, which is itself nothing else than the doctrine of the lever in its more general form.
>
> (Mach 1919, p. 14)

Stevin appealed to the impossibility of a perpetual motion in his derivation of the law of the inclined plane. But this is a virtuous appeal, according to Mach's account, because it appeals only to experiences that every subject on earth has had, and thus his 'procedure is no error. If an error were contained in it, we should all share it' (Mach 1919, p. 27). If Archimedes had simply rested on the instinctive demonstration of the principles of equilibrium (a and b above), he would not have made an error.

Instead, Archimedes tried to find a *deductive* proof: in it, Archimedes appeals not to instinctive or empirical knowledge, but to another, equivalent general principle. Archimedes assumes, rather than appealing to, 'the

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assumption that the equilibrium-disturbing effect of a weight P at the distance L from the axis of rotation is measured by the product P•L' (Mach 1919, p. 14). Mach remarks:

> It is characteristic, that he will not trust on his own authority, perhaps even on that of others, the easily presented observation of the import of the product P•L, but searches after a further verification of it. Now as a matter of fact we shall not, at least at this stage of our progress, attain to any comprehension whatever of the lever unless we directly *discern* in the phenomena the product P•L as the factor decisive of the disturbance of equilibrium. In so far as Archimedes, in his Grecian mania for demonstration, strives to get around this, his deduction is defective. But regarding the import of P•L as given, the Archimedean deductions still retain considerable value, in so far as the modes of conception of different cases are supported the one on the other, in so far as it is shown that one simple case contains all others, in so far as the same mode of conception is established for all cases.
>
> (Mach 1919, pp. 18–19)

Mach's objection to Archimedes is not merely that Archimedes' reasoning is circular. It is that Archimedes' reasoning is *deductive*, and not a demonstration from instinctive knowledge. Mach disagrees with Archimedes on the *possible ways of demonstrating* the law of the lever: Mach is effectively arguing not that Archimedes has begged the question, but that the knowledge that had been gained by Archimedes' time about the law of the lever and about the centre of gravity were effectively the same, and that both arise from a form of 'instinctive knowledge', resting in part on the symmetry of our own bodies, to which Archimedes did not want to appeal.

The law of the lever and the principle of the centre of gravity formulate instinctive knowledge so that we are able to observe, 'in so far as the same mode of conception is established for all cases', that all cases of measurements of equilibrium have similar features. This form of reasoning, for Mach, does not support a form of abstraction to a priori principles that support deductive reasoning. Rather, it supports the use of these principles to promote the *completeness* and *continuity* of experience and observation, so that our experience can be more informative. All this, in turn, promotes the economy of science: 'The function of science, as we take it, is to replace experience' (Mach 1919, pp. 489–490). Elsewhere in *The Development of Mechanics*, Mach makes very illuminating remarks on this score:

> The most important result of our reflections is that *precisely the apparently simplest mechanical principles are of a very complicated character, that these principles are founded on uncompleted experiences, nay on experiences that never can be fully completed, that practically, indeed, they are sufficiently secured, in view of the tolerable stability of our*

> *environment, to serve as the foundation of mathematical deduction, but that they can by no means themselves be regarded as mathematically established truths but only as principles that not only admit of constant control by experience but also require it.*
>
> (Mach 1919, pp. 237–238, emphasis in original)

Scientific principles are ways of gathering experiences, and finding orderings in them, to show that some can be represented as functionally equivalent to others (see the discussion above of the method of differences). The laws and principles of mechanics are formal statements of previous experiences that have revealed functional relationships between experiences. Those relationships are not complete. They can be used 'as the foundation of mathematical deduction', but they can be used only within the experimental method, which provides '*constant control by experience*'.

These principles are not axioms or rules that have universal validity or applicability, nor can their truth be finally established, since they are founded on experiences that in principle can never be completed. However, they can be productive in the sense that they allow for the goal of the completeness of science, within a reasonable human lifespan:

> We must admit ... that there is no result of science which in point of principle could not have been arrived at wholly without methods. But, as a matter of fact, within the short span of a human life and with man's limited powers of memory, any stock of knowledge worthy of the name is unattainable except by the greatest mental economy. Science itself, therefore, may be regarded as a minimal problem, consisting of the completest possible presentment of facts with the least possible expenditure of thought.
>
> (Mach 1919, pp. 489–490)

The above quotation is emblematic of one reading of Mach on economy: that the principles of the economy of science are only intended to make the stunning breadth of scientific experience and knowledge available in more concise form.

But, as should be evident from the above, that is not the only aim of Mach's economy of science. Archimedes' derivation of the law of the lever is 'economical' in this sense, but Archimedes' understanding of it does not contribute to Machian economy. As Archimedes uses the principle, it does not promote the completeness or coherence of our experience, but rather introduces an element that Archimedes believes to be outside our experience, a speculative principle. Mach argues that, instead, we should take the principle to be formulated in a way that shows how it arises from experience and indeed from instinctive knowledge.[9]

[9] Richard Staley's comments were extremely helpful in clarifying the reading in this passage.

The challenge for readings of Mach's economy of science is to explain how economical reasoning can be productive if it consists only in the formulation of knowledge that already existed. One way to explain this is the usual interpretation of 'biological-economical': people find the formal principles or rules describing experience and scientific knowledge that are the most efficient, given our environment and our physiology.

Machian economy is not merely a way to promote efficiency, but promotes distinct epistemic goals as well, including the completeness and continuity of experience and knowledge and the ability to relate facts in one area of science to facts in another. Interpretations of Machian economy should be developed in a way that demonstrates how to work effectively in science using economic principles and methods.

One such way draws on the work of Herbert Breger, who argues that 'mathematical progress' can take place 'by formally recognizing a know-how which existed before' (Breger 2000, p. 224). Breger takes the approach of looking at textbooks and handbooks from previous mathematical traditions (what Ludwik Fleck calls 'vademecum science' and Thomas Kuhn calls the 'context of pedagogy'). Without making a systematic study, he analyses a number of important cases, including Christiaan Huygens, Felix Klein's *Erlangen* programme, and a number of textbooks on calculus and group theory.

Breger notes a trend towards increasing 'abstraction' when moving from the nineteenth to the twentieth century. Klein himself, on Breger's reading, did not see much point in abstraction: 'Later on, in his *Geschichte der Mathematik im 19. Jahrhundert*, Felix Klein mentions the three well-known axioms (associativity, existence of an inverse element, existence of a neutral element), but just for the sake of completeness; reference to them is considered to be an unnecessarily abstract approach' (Klein 1926, pp. 335–336; Breger 2000, p. 224). On Breger's reading, Klein's work on group theory grew from *working with* groups concretely. The more abstract statements about axioms, permutations, transformation rules, and so on are useful only insofar as they allow us to work more effectively with those groups.

Breger's paper does not reach any particular systematic conclusion, nor is it meant to do so. It is meant as a historical intervention to explain that the contemporary structures of abstraction in mathematics were constructed, accompanying a shift in the way mathematics is taught and practiced. Klein emphasised working directly with groups and group actions, and on the acquisition of something like what Mach calls 'instinctive knowledge'. On Breger's account, those who later formalised Klein's programme replaced that instinctive knowledge with abstract, speculative principles that worked more seamlessly with the mathematical methods that took over the field during that time.

The key point is that, for Mach as for Klein, abstract structures or principles are of use insofar as they allow us to grasp the instinctive or tacit knowledge (to use Michael Polanyi's later term) available to us in mathematical and physical practice. Going beyond what Breger argues, Mach argues that mathematical reasoning allows us to apply that instinctive knowledge by analogy across domains (see, especially, Mach 1903).[10]

We could use a number of accounts of the nature of mathematics to describe the role of these principles or structures:

(1) *Instrumentalism or fictionalism.* We construct structures to respond effectively when working with the domain under study. In this sense, the abstract elements and processes are *constructed* but are not intended to capture any *real* structural elements of the groups under study.
(2) *Structuralism or structural realism.* The structures that are constructed allow for processes and 'objects' to emerge that turn out to be informative about the phenomena we are investigating. The structures themselves turn out to be 'real', in the weaker sense that they are part of a framework for referring to real things and processes, or in the stronger sense that the structures can be proven to correspond to real features of things or processes (including dynamical features). Thus, even though the structures are *constructed*, they nonetheless can be shown to be *real*.
(3) *Abstractionism.* A contemporary extension of Fregean logicism, according to which 'abstraction principles play a crucial role in the proper foundation of arithmetic, analysis, and possibly other areas of mathematics'.[11]

As Gary Hatfield (2018) has noted, Helmholtz's view seems to be consistent with a form of structuralism. I would argue against any such view as being consistent with Mach's account, which points to an interesting distinction between Helmholtz and Mach.

Mach also does not seem to be a thoroughgoing fictionalist, which is where I differ from some readings of Mach's economy of science. Mach does *not* argue that we may construct any principles we like as long as they promote the more efficient or concise formulation of scientific data or facts. To say so is to ignore any number of passages in Mach, including those emphasised by Staley (2013 and this volume), in which Mach draws substantially from his research on physiology to back up his physical reasoning, even the reasoning that influenced Einstein's theory of general relativity. A typical passage is

[10] Thanks are due to Richard Staley for pressing for a clarification of this passage.
[11] Ebert and Rossberg (2016, p. 5). Also see other chapters in this volume and in Cook (2007).

found in the 1903 essay 'Space and Geometry from the Point of View of Physical Inquiry':

> Our notions of space are rooted in our physiological constitution. Geometric concepts are the product of the idealisation of physical experiences of space. Systems of geometry, finally, originate in the logical classification of the conceptual materials so gathered. All three factors have left their indubitable traces in modern geometry. Epistemological inquiries regarding space and geometry accordingly concern the physiologist, the psychologist, the physicist, the mathematician, the philosopher, and the logician alike, and they can be gradually carried to their definitive solution only by the consideration of the widely disparate points of view which are here offered.
>
> (Mach 1903, p. 1)

This passage makes two points key regarding Mach on economy. First, Mach sees the 'economy' of science as partly being like a *managed* economy in a society: different sectors of that society may engage in science, and their 'widely disparate points of view' all contribute to the solution of scientific problems (see Schabas 2005).

The second point is a significant and underappreciated element in Mach. To him, logic and mathematics are creative and open-ended, not determinative and reductive:

> Supposing a mathematician to have modified tentatively the simplest and most immediate assumptions of our geometrical experience, and supposing his attempt to have been productive of fresh insight, certainly nothing is more natural than that these researches should be further prosecuted, in a purely mathematical interest. Analogues of the geometry we are familiar with, are constructed on broader and more general assumptions for any number of dimensions, with no pretention to being regarded as more than intellectual scientific experiments and with no idea of being applied to reality.
>
> (Mach 1903, p. 27)

While Mach criticises Archimedes for introducing a deductive method when he should have appealed to instinctive knowledge, he speaks approvingly in this essay of multiple systems of geometry, and of 'idealising' and 'schematising' concepts:

> Geometry, accordingly, consists of the application of mathematics to experiences concerning space. Like mathematical physics, it can become an exact deductive science only on the condition of its representing the objects of experience by means of schematising and idealising concepts. Just as mechanics can assert the constancy of masses or reduce the interactions between bodies to simple accelerations only within the limits

> of errors of observation, so likewise the existence of straight lines, planes, the amount of the angle-sum, etc., can be maintained only on a similar restriction. But just as physics sometimes finds itself constrained to replace its ideal assumptions by other more general ones, viz., to put in the place of a constant acceleration of falling bodies one dependent on the distance, instead of a constant quantity of heat a variable quantity, – so a similar procedure is permissible in geometry, when it is demanded by the facts or is necessary temporarily for scientific elucidation.
>
> (Mach 1903, pp. 20–21)

It may seem as if Mach is contradicting himself. He has criticised Archimedes for replacing 'instinctive knowledge' with a speculative principle, and now Mach is allowing for 'idealisation' and of speculation in geometry and even in mechanics.

However, in my view, Mach's account is consistent. Mach allows for idealisations and schematised representations of experiments *if* those methods are used in a way that opens up new avenues for showing how experiences might be related, produced, and demonstrated – and in distinct contexts, including reasoning about atomism, if their abstract, fictional nature is recognised.[12] A fundamental principle of the Machian economy of science is the principle of the continuity of experience, which I earlier described as anti-reductionism: that world-elements can be related to one another in multiple, open-ended ways, which reveal distinct connections between experiences.

Mach's method is to find the connections between observations and experiences that promote scientific knowledge. To do so requires abstaining from the use of merely speculative hypotheses, and replacing them with idealised and schematic instruments that can be used productively to bring new facts and relationships into view. As a result, Mach's economy of science can be seen, with some justification, as a kind of Machian abstractionism, to distinguish it from Fregean abstractionism. The outlines of Machian abstractionism are sketched above and in Patton (2018), and they remain to be filled out by future research into Mach's work on the economy of science.

Conclusion: Abstraction, Pragmatism, and History as Economical Methods

While accounts that focus on the biological and psychological aspects of Mach's economy of science are well taken, Mach's epistemological methods cannot be captured in their entirety without an account of his pragmatism, historicism, and abstractionism.

[12] Thanks are due to Richard Staley for a fruitful amplification of this claim.

María De Paz (2018) reads Mach as a practice-based thinker along the lines of the description of mathematical practice in Ferreirós (2016). José Ferreirós argues that a practice-based epistemology in mathematics focuses on the mathematical agent (the reasoner), and thus is *cognitive*; moreover, an account of mathematical practice is also *pragmatist* and *historical*. De Paz argues, with reason, that this tripartite focus describes Mach's epistemological methods as well.

Ideally, my approach complements De Paz's, contributing an emphasis on the productive tension between the creative moment in Mach and his biological-economical account. Accounts of Mach's methods that emphasise biology focus on ways to characterise the cognitive activities of a scientific reasoner as containing a subjective component, but also as determined by interaction with the environment. For instance, Isabel Serra and Elisa Maia (2018) read Mach as giving the Kantian a priori a biological slant with his account of instinctive knowledge, following on Pojman's (2000, 2011) earlier reading.

My account pulls apart the two strands of instinctive knowledge and abstraction and considers each as making a separate contribution to Machian economy. It is not opposed to biological readings, but does oppose biological *reductionism*. My reading puts particular emphasis on abstraction as part of the history of science.

Mach does not think that an account of our role as embodied, biologically determined subjects of experience exhausts the contribution of the subjective to epistemology or to economy. The problems which scientists take to be important, the history of approaches to those problems, the principles and laws they adopt, and the methods of abstraction they use are all part of the methods of economical reasoning in science. Knowing how to use effective abstraction principles for one's purposes, for instance, can make one's scientific reasoning much more economical.

I take the account above to be friendly to the pragmatic reading in Pietro Gori (2018, 2019) and in Thomas Uebel's chapter for this volume, which emphasise the pragmatism embodied by Mach's reasoning about the economy of science. As Uebel (this volume) observes, 'historical studies for Mach were far from merely antiquarian. Rather, they intended a "critical epistemological enlightenment of the foundations" of scientific disciplines or sub-disciplines (1896/1986, p. 1, trans. amended) and so were essential to sustaining further progress'.

I am very sympathetic to this approach, and I would add merely my emphasis on the role of abstraction principles arising from instinctive knowledge in creative scientific reasoning. Mach's method involves the following claims, adding one to the earlier stock:

> Features of experience, such as its *continuity*, and whether we can provide a coherent *history* of our experience in giving explanations, support efficient orderings of the data of experience.

> Anti-reductionism: there is no one correct or accurate state of affairs to which a complex of world-elements must be reduced. There can be multiple accurate expressions of the content of a complex of world-elements.
>
> Abstractionism: the use of principles and laws to make connections between different experiences, to allow access to distinct presentations of world-elements, and thus to promote the continuity and connectedness of experience supports the economy of science. These principles arise from instinctive knowledge, and history describes how one led to the other.

The new element of 'abstraction' can be used historically to enable the 'critical epistemological enlightenment of the foundations' of scientific disciplines that Uebel emphasises. A history of scientific reasoning ought not be restricted to a history of phenomenological experience, but rather should include an analysis of the principles that were used to introduce orderings into that experience in order to achieve certain goals. Machian abstractionism, pragmatism, and historicism work together in the economy of science.

References

Banks, Erik C. 2003. *Ernst Mach's World Elements: A Study in Natural Philosophy*. Kluwer.

2004. 'The Philosophical Roots of Mach's Economy of Thought', *Synthese* 139: 23–53.

2014. *The Realistic Empiricism of Mach, James, and Russell: Neutral Monism Reconceived*. Cambridge University Press.

Blackmore, John T. 1972. *Ernst Mach: His Work, Life, and Influence*. University of California Press.

Breger, Herbert 2000. 'Tacit Knowledge and Mathematical Progress', in E. Grosholz and H. Breger (eds.), *The Growth of Mathematical Knowledge*. Kluwer, pp. 221–230.

Cook, Roy T. (ed.) 2007. *The Arché Papers on the Mathematics of Abstraction*. Springer.

De Kock, Liesbet 2014. 'Voluntarism in Early Psychology', *History of Psychology* 17: 105–128.

2015. 'Hermann von Helmholtz's Empirico-transcendentalism Reconsidered', *Science in Context* 27: 709–744.

De Paz, María 2018. 'Reconsidering Mach in the Light of the Interplay of Practices', *Revista Portuguesa de Filosofia* 74: 219–246.

Dijksterhuis, E. J. 1970/2012. *Simon Stevin: Science in the Netherlands around 1600*. Springer.

Ebert, Philip and Rossberg, Marcus 2016. 'Introduction', in P. Ebert and M. Rossberg (eds.), *Abstractionism*. Oxford University Press, pp. 3–34.

Edgar, Scott 2013. 'The Limits of Experience and Explanation: F. A. Lange and Ernst Mach on Things in Themselves', *British Journal for the History of Philosophy* 21: 100–121.

Ferreirós, José 2016. *Mathematical Knowledge and the Interplay of Practices*. Princeton University Press.

Feyerabend, Paul K. 1987. *Farewell to Reason*. Verso.

1999. 'Philosophy of Science: A Subject with a Great Past', in P. K. Feyerabend, *Knowledge, Science and Relativism: Philosophical Papers, Volume 3* (ed. J. M. Preston). Cambridge University Press, pp. 127–137.

Gori, Pietro 2018. 'Ernst Mach and Pragmatic Realism', *Revista Portuguesa de Filosofia* 74: 151–172.

2019. 'What Does It Mean to Orient Oneself in Science? On Ernst Mach's Pragmatic Epistemology', in F. Stadler (ed.), *Ernst Mach – Life, Work, Influence* (Vienna Circle Institute Yearbook). Springer, pp. 525–536.

Grattan-Guinness, Ivor 1974. Review of John T. Blackmore, *Ernst Mach: His Work, Life, and Influence*. *British Journal for the History of Science* 7: 75–79.

Hatfield, Gary 1990. *The Natural and the Normative*. MIT Press.

2018. 'Helmholtz and Philosophy', *Journal for the History of Analytical Philosophy* 6: 12–41.

Helmholtz, Hermann von 1867. *Handbuch der physiologischen Optik*. Leopold Voss.

1878. *Die Thatsachen in der Wahrnehmung*. Hirschwald.

Hui, Alexandra 2013. *The Psychophysical Ear: Musical Experiments, Experimental Sounds, 1840–1910*. MIT Press.

Klein, Felix 1926 *Vorlesungen über die Entwicklung der Mathematik im 19*. Jahrhundert. Springer.

Lenin, Vladimir I. 1909/1972 *Materialism and Empirio-Criticism. Critical Comments on a Reactionary Philosophy*, in *Lenin. Collected Works, Volume 14*. Progress Publishers, pp. 17–362.

Mach, Ernst 1903. 'Space and Geometry from the Point of View of Physical Inquiry', *The Monist* 14: 1–32.

1905. *Erkenntnis und Irrtum*. J.A. Barth.

1919. *The Science of Mechanics: A Critical and Historical Account of Its Development*, 2nd English edition, translated from the 1901 3rd German edition by Thomas J. McCormack. Open Court.

1986. *Popular Scientific Lectures*, trans. Thomas J. McCormack. Open Court.

Palmieri, Paolo 2008. 'The Empirical Basis of Equilibrium: Mach, Vailati, and the Lever', *Studies in History and Philosophy of Science* 39: 42–53.

Patton, Lydia 2004. Hermann Cohen's History and Philosophy of Science. Dissertation, McGill University. Available from http://digitool.library.mcgill.ca/R/?func=dbin-jump-full&object_id=85027&local_base=GEN01-MCG02

2018. 'Helmholtz's Physiological Psychology', in S. Lapointe (ed.), *Philosophy of Mind in the Nineteenth Century*. Routledge, pp. 96–116.

2019. 'New Water in Old Buckets: Hypothetical and Counterfactual Reasoning in Mach's Economy of Science', in F. Stadler (ed.), *Ernst Mach – Life, Work, Influence* (Vienna Circle Institute Yearbook). Springer, pp. 333–348.

Pojman, Paul 2000. *Ernst Mach's Biological Theory of Knowledge*. Dissertation, Indiana University.

2011. 'The Influence of Biology and Psychology upon Physics: Ernst Mach Revisited', *Perspectives on Science* 19: 121–135.

Schabas, Margaret 2005. *The Natural Origins of Economics*. University of Chicago Press.

Seaman, Francis 1975. Review of John T. Blackmore, *Ernst Mach: His Work, Life, and Influence*. *Journal of the History of Philosophy* 13: 273–276.

Serra, Isabel and Maia, Elisa 2018. 'Looking for Routes in Mach's Epistemology', *Revista Portuguesa de Filosofia* 74: 173–196.

Staley, Richard 2013. 'Ernst Mach on Bodies and Buckets', *Physics Today* 66: 42–47.

2019. 'Revisiting Einstein's Happiest Thought', in F. Stadler (ed.), *Ernst Mach – Life, Work, Influence* (Vienna Circle Institute Yearbook). Springer, pp. 349–366.

Stein, Howard 1967. 'Newtonian Space–Time', *Texas Quarterly* 10: 174–200.

1977. 'Some Philosophical Prehistory of General Relativity', in J. Earman, C. Glymour, and J. Stachel (eds.), *Foundations of Space–Time Theories*. University of Minnesota Press, pp. 3–49.

Stevin, Simon 1586. De beghinselen der weeghconst. Druckerye van Christoffel Plantijn.

Tracz, Brian 2018. 'Helmholtz on Perceptual Properties', *Journal for the History of Analytical Philosophy* 100: 64–78.

van Dyck, Maarten 2009. 'On the Epistemological Foundations of the Law of the Lever', *Studies in History and Philosophy of Science* 40: 315–318.

Wulz, Monika 2015. 'Gedankenexperimente im ökonomischen Überschuss. Wissenschaft und Ökonomie bei Ernst Mach', *Berichte zur Wissenschaftsgeschichte* 38: 59–76.